

Some Additional Theory in Differential Equations

Now we need some results from the theory of linear differential equations.

Fact 1: EVERY solution of $du/dt = Au$ has the form $u = c_1u_1 + c_2u_2$ provided u_1 and u_2 are linearly independent solutions (assuming A is 2×2 and u is in \mathbb{R}^2).

Fact 2: The c_i are determined by specifying an “initial condition” for u (i.e, for v and w):

$$u(0) = \begin{bmatrix} v(0) \\ w(0) \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix} \quad (\text{say})$$

For example:

$$\begin{bmatrix} 8 \\ 5 \end{bmatrix} = c_1u_1(0) + c_2u_2(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$