Areas in 3D

We conclude that

$$\text{area } A = \left|\det\begin{bmatrix} | & | \\ B & a \\ | & | \end{bmatrix}\right| = \left|\det\begin{bmatrix} | & | \\ b-p & a \\ | & | \end{bmatrix}\right| = \left|\det\begin{bmatrix} | & | \\ b & a \\ | & | \end{bmatrix}\right|$$

the last step since p is a multiple of a and so can be eliminated by a column operation.

A nice application of the volume in 3D formula is the task of finding the area of a parallelogram in 3D formed by two vectors a and b. Simply find a unit vector c that is orthogonal to a and b. Then the area of the parallelogram is the same as the volume of the solid defined by a, b and c:

$$\text{area } A = \left|\det\begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix}\right|, c \perp a, c \perp b, \|c\| = 1$$