An Example

The diagonal elements are cofactor expansions along each row of A and hence all are $\det A$. The off diagonal entries are all zero. For example, the 2,1 entry is

$$a_{21}C_{11} + a_{22}C_{12} + \cdots + a_{2n}C_{1n} = \begin{vmatrix} a_{21} & a_{22} & \cdots & a_{2n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0$$

since two rows are the same. We conclude that $AC^T = (\det A)I$ and the theorem follows.

Ex:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$