General Cofactor Expansions

We simply need to evaluate the determinant of M_{ij}. First move row i to row 1 and shift the first $i-1$ rows down one position. This involves i row exchanges and so brings in a factor $(-1)^i$. Now move column j to column 1 and shift the first $j-1$ columns one position to the right. This involves j column exchanges and so brings in a factor $(-1)^j$. The resulting matrix now has the first row of I as its first row and so our previous result applies:

$$|M_{ij}| = (-1)^{i+j} \det(A \text{ with row } i \text{ and column } j \text{ removed}) = C_{ij}$$

Thus we have the **cofactor expansion of** $\det A$ **along the** ith **row**:

$$|A| = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}$$

Note: Since A and A^T have the same determinant, we can compute $\det A$ by a cofactor expansion along any row **or** any column! It is wise to use a row or column that has lots of zero entries!