Some Observations

We show now that similar expansions apply for general $n \times n$ matrices. Let B be an $(n-1) \times (n-1)$ matrix and consider the $n \times n$ matrix

$$A = \left[\begin{array}{c|ccc} 1 & 0 & \cdots & 0 \\ \hline b_1 & & & \\ \vdots & & B & \\ b_{n-1} & & & \end{array}\right]$$

whose first row is the first row of the identity matrix. If we perform G-E, the b_i's will become zeros and B will be reduced to its row echelon form. By PROP 7 the determinant of the echelon form of B is the product of its diagonal entries. If we multiply this product by 1 we get the determinant of A. Thus we have shown that $\det A = \det B$.