

# Cofactor Expansions

The six term expression we have just found can be written in a different and suggestive form:

$$\begin{aligned} |A| &= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) \\ &\quad + a_{13}(a_{21}a_{32} - a_{22}a_{31}) \\ &= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \end{aligned}$$

**Definition:** The cofactor of position  $ij$  in  $A$  is the number

$$C_{ij} = (-1)^{i+j} \det(A \text{ with row } i \text{ and column } j \text{ removed})$$

In terms of this definition our expansion above is

$$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

This is the **cofactor expansion of  $\det A$  along the first row.**