Permutation Formula for the Determinant

Let $Pijk$ be the permutation matrix with rows e_i, e_j, and e_k from I. Then each term in the sum has the form

$$a_{1i}a_{2j}a_{3k}\left|P_{ijk}\right|$$

where (i, j, k) is some permutation of $(1, 2, 3)$. That is

$$\det A = \sum_{\text{all perms } (i,j,k) \text{ of } (1,2,3)} a_{1i}a_{2j}a_{3k}\left|P_{ijk}\right|$$

This generalizes to $n \times n$ matrices in the natural way.

Of course, we know from DEF PROP 2 how to evaluate determinants of permutation matrices:

$$\left|P_{ijk}\right| = \begin{cases} 1 & \text{for an even number of row exchanges} \\ -1 & \text{for an odd number of row exchanges} \end{cases}$$