A Similar 3 x 3 Determinant Expansion

Now look at the 3×3 case:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix} = a_{11}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + a_{12}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} + a_{13}\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

and so linear expansions of $|A|$ along rows 1, 2 and 3 produces 27 determinants. All but 6 of these have zero columns (like above) and we are left with

$|A| =$

$$a_{11}a_{22}a_{33}\underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{\text{det of } I} + a_{12}a_{23}a_{31}\underbrace{\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}}_{\text{double perm}} + a_{13}a_{21}a_{32}\underbrace{\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}}_{\text{double perm}}$$

$$+ a_{11}a_{23}a_{32}\underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}_{\text{single perm}} + a_{12}a_{21}a_{33}\underbrace{\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{\text{single perm}} + a_{13}a_{22}a_{31}\underbrace{\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}}_{\text{single perm}}$$