Some Additional Properties

Then

$$|UB| = \begin{vmatrix} - & d_1 b_1 + u_{12} b_2 + u_{13} b_3 & - \\ - & d_2 b_2 + u_{23} b_3 & - \\ - & d_3 b_3 & - \end{vmatrix} = \underbrace{\begin{vmatrix} - & d_1 b_1 + 0 b_2 + 0 b_3 & - \\ - & d_2 b_2 + 0 b_3 & - \\ - & d_3 b_3 & - \end{vmatrix}}_{\text{PROP 5}}$$

$$= \underbrace{d_1 d_2 d_3 \begin{vmatrix} - & b_1 & - \\ - & b_2 & - \\ - & b_3 & - \end{vmatrix}}_{\text{DEF PROP 2 \& 3}} = d_1 d_2 d_3 |B| \underbrace{= |U| \times |B|}_{\text{PROP 7}}$$

Now recall the factorization $PA = LU$. P performs row exchanges (multiplying det by -1 each time) and L performs row operations (that don't change det). Thus

$$|AB| = \pm |PAB| = \pm |LUB| = \pm |UB| = \pm |U| \times |B| = |A| \times |B|$$