Some Additional Properties

PROP 8: A is invertible if and only if $\det A \neq 0$
Proof: Let U be the row echelon form of A. By DEF PROP 2 and PROP 5, $\det A = \pm \det U$. But A is singular only when U has a zero row, and the conclusion follows.

PROP 9: $\det(AB) = \det A \times \det B$
Proof: We consider just the 3×3 case. First we show that $\det(UB) = \det U \times \det B$ if U is upper triangular. If U has a zero row, so does UB, so both have a zero determinant. Otherwise assume

$$U = \begin{bmatrix} d_1 & u_{12} & u_{13} \\ 0 & d_2 & u_{23} \\ 0 & 0 & d_3 \end{bmatrix} \text{ where } d_1 d_2 d_3 \neq 0, B = \begin{bmatrix} - & b_1 & - \\ - & b_2 & - \\ - & b_3 & - \end{bmatrix}$$