Some Additional Properties

PROP 7: If A is triangular, then $\det A =$ product of the diagonal entries.
Proof: We illustrate this for 3×3 matrices:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}$$

Here is the method:

$$|A| = \underbrace{-\begin{vmatrix} 0 & 0 & a_{33} \\ 0 & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix}}_{\text{DEF PROP 2}} \underbrace{= -a_{33}\begin{vmatrix} 0 & 0 & 1 \\ 0 & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix}}_{\text{DEF PROP 3}} \underbrace{= a_{33}\begin{vmatrix} 0 & a_{22} & a_{23} \\ 0 & 0 & 1 \\ a_{11} & a_{12} & a_{13} \end{vmatrix}}_{\text{DEF PROP 2}}$$