The Determinant

Definition: The determinant of a square matrix A, denoted $\det A$ or $|A|$, is a function of the entries of A with the property:

$$A \text{ is invertible if and only if } \det A \neq 0$$

So what does such a function look like? Recall that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

We define

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

and so for 2×2 matrices, $\det A \neq 0 \iff A$ is invertible.

For general $n \times n$ matrices three defining properties completely characterize the determinant:

DEF PROP 1: $\det I = 1$

DEF PROP 2: $\det A = -\det B$ if B is A with one row interchange

DEF PROP 3: $\det A$ is a linear function of its first row