Gram-Schmidt (complete)

Step 5: Normalize C

$$\|C\|^2 = \int_{-1}^{1} \left(t^2 - \frac{1}{3} \right)^2 dt = ... = \frac{8}{45}$$

$$q_3(t) = \frac{\sqrt{5 \times 9}}{2\sqrt{2}} t^2 - \frac{1}{3} = \frac{\sqrt{5}}{2\sqrt{2}}(3t^2 - 1)$$

Recall our original venture: find the quadratic function that best approximates $f(t)$ on $[-1, 1]$, and our proprosal is to project f onto P_2. Now that we have an orthonormal basis of P_2, that approximation is

$$f_{approx}(t) = (q_1, f)q_1(t) + (q_2, f)q_2(t) + (q_3, f)q_3(t)$$

$$= c_1 \frac{1}{\sqrt{2}} + c_2 \sqrt{\frac{3}{2}} t + c_3 \frac{\sqrt{5}}{2\sqrt{2}}(3t^2 - 1)$$