

Gram-Schmidt (continuing)

Step 3: Normalize B

$$\|B\|^2 = \int_{-1}^1 t^2 dt = \left. \frac{t^3}{3} \right|_{-1}^1 = \frac{2}{3} \Rightarrow q_2(t) = \sqrt{\frac{3}{2}}t$$

Step 4: Remove from c its projections onto q_1 and q_2

$$C(t) = c(t) - (q_1, c)q_1(t) - (q_2, c)q_2(t)$$

$$(q_1, c) = \int_{-1}^1 q_1(t)c(t)dt = \int_{-1}^1 \frac{1}{\sqrt{2}}t^2 dt = \left. \frac{t^3}{3\sqrt{2}} \right|_{-1}^1 = \frac{2}{3\sqrt{2}}$$

$$(q_2, c) = \int_{-1}^1 q_2(t)c(t)dt = \int_{-1}^1 \sqrt{\frac{3}{2}}t \times t^2 dt = \left. \sqrt{\frac{3}{2}} \frac{t^4}{4} \right|_{-1}^1 = 0$$

$$C(t) = t^2 - \frac{2}{3\sqrt{2}} \times \frac{1}{\sqrt{2}} = t^2 - \frac{1}{3}$$