Orthogonal Polynomials via Gram-Schmidt

How can we find an orthonormal basis of P_2? We will start with the standard basis $\{1, t, t^2\}$ and apply Gram-Schmidt, which simply requires an inner product (and has nothing to do specifically with column vectors).

Set

$$a(t) = 1, b(t) = t, c(t) = t^2$$

Step 1: Normalize a

$$\|a\|^2 = \int_{-1}^{1} a(t)^2 dt = \int_{-1}^{1} dt = 2 \Rightarrow q_1(t) = \frac{1}{\sqrt{2}}$$

Step 2: Remove from b its projection onto q_1

$$B(t) = b(t) - (q_1, b)q_1(t)$$

$$(q_1, b) = \int_{-1}^{1} q_1(t)b(t)dt = \int_{-1}^{1} \frac{1}{\sqrt{2}} t dt = \left. \frac{t^2}{2\sqrt{2}} \right|_{-1}^{1} = 0, B(t) = t$$