Approximating Functions by Polynomials

Question: Given a function $f(t)$ on $[-1, 1]$, what is the quadratic function $a_0 + a_1 t + a_2 t^2$ that most closely approximates f (in some sense)?

Since P_2 is a subspace of the space of all functions defined on $[-1, 1]$, one solution of the problem above comes from projecting f onto P_2. This will give us the best approximation in the sense of least squares error relative to:

$$\|f\|^2 = \int_{-1}^{1} f(t)^2 dt$$

So far we principally know how to project onto the column space of a matrix (i.e. every subspace of $\mathbb{R}^n$ is the column space of some matrix). But here we have polynomials. Let us also note that we know how to project onto any subspace with an orthonormal basis. And this requires only an inner product (and has nothing to do specifically with column vectors). Does this observation help?