Fourier Series and Fourier Coefficients

The orthonormal set above spans a large class of functions that are periodic of period 2π. We should be able to write any such function $f(t)$ as a linear combo of the members of the orthonormal set. This gives the expansion

$$f(x) = a_0 \frac{1}{\sqrt{2\pi}} + a_1 \frac{\cos t}{\sqrt{\pi}} + a_2 \frac{\cos 2t}{\sqrt{\pi}} + \cdots + b_1 \frac{\sin t}{\sqrt{\pi}} + b_2 \frac{\sin 2t}{\sqrt{\pi}} + \cdots$$

We call this the **Fouries Series** of f. To compute it we need to find the coefficients. But the coefficients relative to an orthonormal set are simple inner products of f with the orthonormal vectors:

$$a_0 = (\frac{1}{\sqrt{2\pi}}, f), a_n = (\frac{\cos nt}{\sqrt{\pi}}, f), b_n = (\frac{\sin nt}{\sqrt{\pi}}, f)$$

More explicitly:

$$a_0 = \int_0^{2\pi} \frac{1}{\sqrt{2\pi}} f(t) dt, a_n = \int_0^{2\pi} \frac{\cos nt}{\sqrt{\pi}} f(t) dt, b_n = \int_0^{2\pi} \frac{\sin nt}{\sqrt{\pi}} f(t) dt$$

These are the classical Fourier coefficients.