Example (continued)

$$\left.\begin{aligned}(\cos nt, \cos mt) &= \int_0^{2\pi} \cos nt \cos mt dt = 0 \text{ if } m \neq n \\ (\sin nt, \sin mt) &= \int_0^{2\pi} \sin nt \sin mt dt = 0 \text{ if } m \neq n \\ (\cos nt, \sin mt) &= \int_0^{2\pi} \cos nt \sin mt dt = 0\end{aligned}\right\} \text{Calc II Trig Integrals}$$

We conclude that all these functions are mutually orthogonal! Let's normalize them:

$$\|1\|^2 = \int_0^{2\pi} 1^2 dt = 2\pi$$

$$\|\cos nt\|^2 = \int_0^{2\pi} \cos^2 nt dt = \cdots = \pi = \|\sin nt\|^2$$

We conclude that

$$\left\{ \frac{1}{\sqrt{2\pi}}, \frac{\cos t}{\sqrt{\pi}}, \frac{\sin t}{\sqrt{\pi}}, \frac{\cos 2t}{\sqrt{\pi}}, \frac{\sin 2t}{\sqrt{\pi}}, \cdots \right\}$$

is an orthonormal set in the space of functions on $[0, 2\pi]$.