

## Example

**Definition:** The **inner product** and **norm** (size) of functions on interval  $[a, b]$  are

$$(f, g) = \int_a^b f(t)g(t)dt, \|f\|^2 = (f, f) = \int_a^b f(t)^2 dt$$

**Ex:** Demonstrate that the collection of functions  $\{1, \cos t, \sin t, \cos 2t, \sin 2t, \dots\}$  is mutually orthogonal as functions on  $[0, 2\pi]$ .

Let's compute some inner products:

$$(1, \cos nt) = \int_0^{2\pi} 1 \times \cos ntdt = \frac{1}{n} \sin nt \Big|_0^{2\pi} = 0 \quad (\text{orthogonal!})$$

$$(1, \sin mt) = \int_0^{2\pi} 1 \times \sin mtdt = -\frac{1}{m} \cos mt \Big|_0^{2\pi} = 0 \quad (\text{orthogonal!})$$