Inner Product in Function Space

A better measure of size is given by the *average* of this sum. If in addition we let n tend to ∞, we obtain

$$\|f\|^2 = \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n} f(t_i)^2 = \lim_{n\to\infty} \frac{1}{b-a}\sum_{i=0}^{n} f(t_i)^2 \Delta t_i = \frac{1}{b-a}\int_a^b f(t)^2 dt$$

A similar calculation with functions f and g gives

$$(f,g) = \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n} f(t_i)g(t_i) = \lim_{n\to\infty} \frac{1}{b-a}\sum_{i=0}^{n} f(t_i)g(t_i)\Delta t_i$$
$$= \frac{1}{b-a}\int_a^b f(t)g(t)dt$$

In general the factor $\frac{1}{b-a}$ is not needed to define a measure of size and inner product of functions, so we will drop it.