Example (continued)

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, q_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

We conclude that the matrix

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

is orthogonal.

While our example here involved three basis vectors in $\mathbb{R}^3$, the vectors could be in a larger space such as $\mathbb{R}^4$ or $\mathbb{R}^7$ and just span a subspace of this larger space. In this case the Q you get is not square (and hence not orthogonal), but it does have orthonormal columns (and hence $Q^T Q = I$).