Orthogonal Matrices

Definition: A matrix Q is **orthogonal** if it is square and its columns are orthonormal.

Ex:

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

Note: $Q^TQ = I$ (with Q square) implies that $Q^{-1} = Q^T$. It follows also that $QQ^T = I$ and this shows that the rows of Q are also orthonormal! (Do you see this in the example above?)

Ex: $Q_\theta = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$, $c = \cos\theta, s = \sin\theta$, is a θ rotation of $\mathbb{R}^2$. Note that

$$Q_\theta^{-1} = \frac{1}{c^2+s^2}\begin{bmatrix} c & s \\ -s & c \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}^T = Q_\theta^T$$