Factorization (continued)

This implies that the matrix A with a, b, c as columns has the form

$$A = \left[Q \begin{bmatrix} (q_1, a) \\ 0 \\ 0 \end{bmatrix}, Q \begin{bmatrix} (q_1, b) \\ (q_2, b) \\ 0 \end{bmatrix}, Q \begin{bmatrix} (q_1, c) \\ (q_2, c) \\ (q_3, c) \end{bmatrix} \right]$$

$$= Q \begin{bmatrix} (q_1, a) & (q_1, a) & (q_1, c) \\ 0 & (q_2, b) & (q_2, c) \\ 0 & 0 & (q_3, c) \end{bmatrix} = QR$$

Theorem: Let A be $m \times n$ with linearly independent columns. Then there is an $n \times n$ upper triangular invertible matrix R and an $m \times n$ matrix Q with orthonormal columns such that

$$A = QR$$