Matrices With Orthonormal Columns

We have already introduced the idea of an orthonormal basis (recall what this is). We want to explore now why such bases are useful and also how we can find one.

We begin with an observation. Let $q_1, q_2, ..., q_k$ be mutually orthogonal unit vectors in $\mathbb{R}^n$ (where $k \leq n$). This means that

$$(q_i, q_j) = \delta_{ij} = \begin{cases} 0 & \text{if} \quad i \neq j \\ 1 & \text{if} \quad i = j \end{cases}$$

Let Q denote the matrix having the q_i's as columns. Then another way to write the result above is

$$Q^T Q = \begin{bmatrix} - & q_1^T & - \\ \vdots & \vdots & \vdots \\ - & q_k^T & - \end{bmatrix} \begin{bmatrix} | & \cdots & | \\ q_1 & \cdots & q_k \\ | & \cdots & | \end{bmatrix}$$