Projections and Orthonormal Bases

Are projections any simpler when we have an orthonormal basis of our subspace S? Let $\{q_1, q_2, ..., q_r\}$ be such a basis and let A have the q_i's as columns. Then the ij entry of $A^T A$ is just (q_i, q_j) (think about it a bit and you will see why). But this inner product is 0 if $i \neq j$ and 1 if $i = j$. That is, $A^T A = I$. Then the projection matrix is just $P = AA^T$.

Before going further, let us recall projection matrices that project onto a line given by a vector a and observe the simplification that occurs when a is a unit vector q:

$$P = \frac{1}{\|a\|^2} aa^T = \frac{1}{\|q\|^2} qq^T = qq^T \quad \text{since} \quad \|q\| = 1$$