Some Additional Facts About Projections

Remember: every subspace S of $\mathbb{R}^n$ is the column space $C(A)$ of any matrix A whose columns are a basis of S. Since we know from the above theory how to project vectors onto a column space, we therefore know how to project vectors onto a general subspace S.

Orthogonal Decompositions Revisited: Recall

$\mathbb{R}^n = S \oplus S^\perp$ means $v = v_S + v_{S^\perp}$ uniquely with v_S in S and $v_{S^\perp}$ in $S^\perp$

Another way to write this is interms of the matrix P_S projecting onto S and the matrix $P_{S^\perp}$ projecting onto $S^\perp$:

$$v = v_S + v_{S^\perp} \text{ where } v_S = P_S v \text{ and } v_{S^\perp} = P_{S^\perp} v, \; P_{S^\perp} = I - P_S$$