The Projection Matrix

As we did with the projection of b onto a vector a, we wonder whether we can write $p = Pb$ for some matrix P:

$$p = A\hat{x} = A((A^TA)^{-1}A^Tb) = (A(A^TA)^{-1}A^T)b$$

and so

$$p = Pb \text{ where } P = A(A^TA)^{-1}A^T$$

(is P exactly $\frac{1}{||a||^2}aa^T$ if A has a as its only column?)

The matrix P above is the **projection matrix onto the column space of the matrix** A.

Note: You should verify for yourself that this P satifies the two projection properties $P^2 = P$ and $P^T = P$ (recall that A^TA is always a symmetric matrix).