Example (continued)

We see that the normal equations and their solution are:

$$\underbrace{\begin{bmatrix} 2 & 5 \\ 5 & 13 \end{bmatrix}}_{A^T A} \hat{x} = \underbrace{\begin{bmatrix} 9 \\ 23 \end{bmatrix}}_{A^T b} \Rightarrow \hat{x} = \underbrace{\begin{bmatrix} 2 & 5 \\ 5 & 13 \end{bmatrix}^{-1}}_{(A^T A)^{-1}} \underbrace{\begin{bmatrix} 9 \\ 23 \end{bmatrix}}_{A^T b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Therefore the closest vector to b in the column space of A and the error involved are:

$$p = A\hat{x} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$$

$$\text{squared error} = \|e\|^2 = \left\| \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} \right\|^2 = \left\| \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\|^2 = 1$$

The relative error is then $\|e\| \,/\, \|b\| = 1/\sqrt{42} \approx 15.43\%$.