An Example

Note: These are precisely the equations we obtained in the last part of the notes by calculus and therefore their solution $\hat{x}$ represents the best approximate solution for x of $Ax = b$ in the sense of **least squared error**.

Ex:

$$
\begin{array}{rcl}
x_1 + 2x_2 & = & 4 \\
x_1 + 3x_2 & = & 5 \\
0x_1 + 0x_2 & = & 1
\end{array}
\Rightarrow A = \left[\begin{array}{cc} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{array} \right], b = \left[\begin{array}{c} 4 \\ 5 \\ 1 \end{array} \right]
\quad \text{(three equations in two unknowns)}
$$

Then

$$
A^T A = \left[\begin{array}{ccc} 1 & 1 & 0 \\ 2 & 3 & 0 \end{array} \right] \left[\begin{array}{cc} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{array} \right] = \left[\begin{array}{cc} 2 & 5 \\ 5 & 13 \end{array} \right]
$$

$$
A^T b = \left[\begin{array}{ccc} 1 & 1 & 0 \\ 2 & 3 & 0 \end{array} \right] \left[\begin{array}{c} 4 \\ 5 \\ 1 \end{array} \right] = \left[\begin{array}{c} 9 \\ 23 \end{array} \right]
$$