Properties of Projection Matrices

Thus, since $\|a\| = 1$ in this case,

$$P_\theta = aa^T = \begin{bmatrix} c \\ s \end{bmatrix} \begin{bmatrix} c & s \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Later we will define more general projection matrices, but all have two essential properties:

Property 1: $P^2 = P$ (why is this reasonable?)

Verification: $P^2 = \frac{1}{\|a\|^2} aa^T \frac{1}{\|a\|^2} aa^T = \frac{1}{\|a\|^4} a \underbrace{a^T a}_{\|a\|^2} a^T = \frac{1}{\|a\|^2} aa^T = P$

Property 2: $P^T = P$

Verification: $P^T = (\frac{1}{\|a\|^2} aa^T)^T = \frac{1}{\|a\|^2} (aa^T)^T = \frac{1}{\|a\|^2} aa^T = P$