An Example

Definition: The matrix

$$P = \frac{1}{\left\|\boldsymbol{a}\right\|^2} \boldsymbol{a} \boldsymbol{a}^T$$

is the matrix that projects vectors (by matrix multiplication) onto the line formed by *a*. Here is a check. Since $(a, b) = a^T b$,

$$Pb = \frac{1}{\|a\|^2} aa^T b = \frac{1}{\|a\|^2} a(a, b) = \frac{(a, b)}{\|a\|^2} a(a, b)$$

Ex: Let's find the matrix P_{θ} that projects vectors in the plane onto a line at angle θ . Note what *a* is in this case:

$$\begin{array}{c} v_2 \\ \bullet \\ \bullet \\ e_1 \end{array} = \begin{bmatrix} c \\ s \end{bmatrix} \quad a = \begin{bmatrix} c \\ s \end{bmatrix} \quad \text{where} \quad \begin{array}{c} c = \cos \theta \\ s = \sin \theta \end{array}$$