# Projection of a Vector Onto a Line

So how do we determine $x$ so that $p = xa$ is the orthogonal projection/best approximate solution?

**Method 1:** Impose orthogonality
Assume that $e = b - p = b - xa \perp a$ as suggested by the figure. This implies that

$$0 = (a, e) = (a, b - xa) = (a, b) - x \|a\|^2$$
$$\Rightarrow x = \frac{(a, b)}{\|a\|^2} \quad \text{and} \quad p = \frac{(a, b)}{\|a\|^2} a$$

**Method 2:** Minimize $\|e\|$ with respect to $x$ using calculus

$$f(x) = \|e\|^2 = (b - xa, b - xa) = \|b\|^2 - 2x(a, b) + x^2 \|a\|^2$$
$$0 = f'(x) = -2(a, b) + 2x \|a\|^2$$
$$\Rightarrow x \quad \text{and} \quad p \quad \text{as above}$$