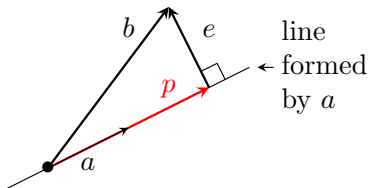


# Inconsistent but Almost Solvable!

What do we do if  $Ax = b$  is inconsistent, i.e.  $b$  is not in the column space  $C(A)$ , but  $b$  is “almost” in  $C(A)$  in some sense? For example, suppose that  $A$  has one column  $a$ . This means that  $x$  is actually a real number and  $Ax = b$  becomes  $xa = b$ : is  $b$  a multiple of  $a$ ? Here is a figure:



Let  $p$  be the orthogonal projection of  $b$  onto the line formed by  $a$  and let  $e = b - p$  so that  $\|e\|$  measures the distance between  $b$  and  $p$ . This means that  $xa = p$  is a good approximation to  $xa = b$ , especially when  $\|e\|$  is small. This leads to the topic of **projections** and **approximate solutions of linear systems**.