Example (continued)

If A has the two vectors here as columns, then

$$A^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{\text{G-J}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = -x_3 \\ x_2 = 0 \\ x_4 \text{ arbitrary} \end{array}$$

The "special" solutions come from setting $x_3 = 1, x_4 = 0$ and then $x_3 = 0, x_4 = 1$. This gives the final result:

$$S^{\perp} = \text{Span}\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$