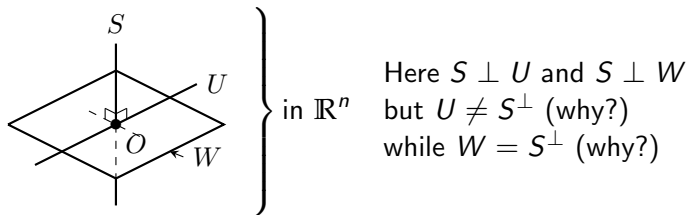## Orthogonal Complements

Definition: Let S be a subspace of a vector space V. The **orthogonal complement** $S^{\perp}$ ('ess perp') of S is the set of all vectors orthogonal to every vector in S.

Here $S \perp U$ and $S \perp W$
but $U \neq S^{\perp}$ (why?)
while $W = S^{\perp}$ (why?)

in $\mathbb{R}^n$

Note: if $(v_1, s) = 0$ and $(v_2, s) = 0$ for every s in S, then
$(c_1 v_1 + c_2 v_2, s) = c_1(v_1, s) + c_2(v_2, s) = c_1 0 + c_2 0 = 0$ for every s in S.
Therefore $S^{\perp}$ is a subspace.