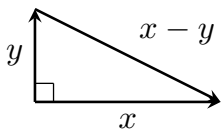# Orthonormal Basis

You might wonder whether this definition of orthogonal agrees with the geometric idea of being "perpendicular." Start with a right triangle and appeal to the Pythagorean Theorem:

$$\underbrace{\|x - y\|^2}_{((x-y),(x-y))} = \|x\|^2 + \|y\|^2$$

$$\underbrace{(x,x)-(x,y)-(y,x)+(y,y)}_{\|x\|^2-2(x,y)+\|y\|^2}$$

$$\implies (x, y) = 0$$

Definition: Vectors $v_1, v_2, ..., v_k$ are called **mutually orthogonal** if

$$(v_i, v_j) = 0 \text{ for all } i \neq j$$

An **orthonormal basis** is a basis whose members are mutually orthogonal and all unit vectors.