The Value of Coordinates

Vectors and linear transformations can at times be rather exotic, but in coordinates they become column vectors and matrices. As such they are easy to program into a computer. But how do you apply T to a vector v in terms of coordinates? If $w = T(v)$ and $[w]_H = y, [v]_F = x, [T]_{HF} = A$, then

$$\begin{aligned} w &= y_1 h_1 + \cdots + y_m h_m = T(x_1 f_1 + \cdots + x_n f_n) \\ &= x_1 T(f_1) + \cdots + x_n T(f_n) \\ &= x_1(a_{11} h_1 + \cdots + a_{m1} h_m) + \cdots + x_n(a_{1n} h_1 + \cdots + a_{mn} h_m) \\ &= (a_{11} x_1 + \cdots + a_{1n} x_n) h_1 + \cdots + (a_{m1} x_1 + \cdots + a_{mn} x_n) h_m \end{aligned}$$

$$\text{i.e. } y_1 = a_{11} x_1 + \cdots + a_{1n} x_n, \ldots, y_m = a_{m1} x_1 + \cdots + a_{mn} x_n$$

$$\text{i.e. } y = Ax$$

In other terms:

$w = T(v) \Longleftrightarrow y = Ax$ i.e multiple the coordinate vector of v by the coordinate matrix of T to get the coordinate vector of w