The Coordinate Matrix of a Linear Transformation

Let $T : V \to W$ be a linear transformation. Let $F = (f_1, ..., f_n)$ be an ordered basis of V (i.e dim $V = n$) and $H = (h_1, ..., h_m)$ be an ordered basis of W (i.e dim $W = m$). We have seen previously that T is completely determined by its values on a basis, i.e. by $T(f_1), ..., T(f_n)$. Each of these is a vector in W and thus uniquely represented in terms of the basis H:

$$\begin{array}{rcl} T(f_1) & = & a_{11}h_1 + \cdots + a_{m1}h_m \\ T(f_2) & = & a_{12}h_1 + \cdots + a_{m2}h_m \\ \vdots & \vdots & \vdots \\ T(f_n) & = & a_{1n}h_1 + \cdots + a_{mn}h_m \end{array}$$

The matrix

$$[T]_{HF} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

is called the **coordinate matrix** of T relative to the bases F and H.