Example (continued)

Thus

$$\begin{bmatrix} 5 & 9 \\ -1 & -1 \end{bmatrix} = 5e_1 + 9e_2 - 1e_3 - 1e_4 \Longrightarrow [\begin{bmatrix} 5 & 9 \\ -1 & -1 \end{bmatrix}]_E = \begin{bmatrix} 5 \\ 9 \\ -1 \\ -1 \end{bmatrix}$$

Now set $F = (f_1, f_2, f_3, f_4)$ where

$$f_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, f_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, f_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, f_4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

This is another basis of $\mathbb{R}^{2\times 2}$ (how would you show this?). Then

$$\begin{bmatrix} 5 & 9 \\ -1 & -1 \end{bmatrix} = \underbrace{2f_1 + 3f_2 + 4f_3 + 5f_4}_{\text{how do you find these numbers?}} \Longrightarrow [\begin{bmatrix} 5 & 9 \\ -1 & -1 \end{bmatrix}]_F = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$