Example (continued)

Let $F = (f_0, f_1, f_2, f_3)$ where

$$f_0(t) = 1, f_1(t) = 1 + t, f_2(t) = 1 + t + t^2, f_3(t) = 1 + t + t^2 + t^3$$

You should verify for yourself that this is another ordered basis of P_3 (linearly independent and spanning P_3 - what follows now is a suggestion of how you do this). Let's find the coordinates of the polynomial v above in this new basis. This means we need to find real numbers c_0, c_1, c_2, c_3 such that

$$\begin{aligned} v(t) &= 2 + 3t + 4t^2 + 5t^3 \\ &= c_0\left(1\right) + c_1\left(1 + t\right) + c_2\left(1 + t + t^2\right) + c_3\left(1 + t + t^2 + t^3\right) \end{aligned}$$

Rearrange the right side by collecting all constant terms, then all linear terms, etc.:

$$\begin{aligned} v(t) &= 2 + 3t + 4t^2 + 5t^3 \\ &= (c_0 + c_1 + c_2 + c_3) + (c_1 + c_2 + c_3)t + (c_2 + c_3)t^2 + c_3 t^3 \end{aligned}$$