An Instructive Examples

Recall from a previous part of these notes that

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{v_1 + v_2}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{v_1 - v_2}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Let E denote the ordered basis (e_1, e_2) of the two columns of the identity $I = I_2$. Also let $F = (f_1, f_2)$ where f_1 has entries 1,1 and f_2 has entries 1,-1. Then the above statement can be written

$$[\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}]_E = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \text{and} \quad [\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}]_F = \begin{bmatrix} \frac{v_1+v_2}{2} \\ \frac{v_1-v_2}{2} \end{bmatrix}$$

When dealing with column vectors, i.e. members of $\mathbb{R}^n$, it is conventional to denote by $E = (e_1, ..., e_n)$ the **standard basis** consisting of the columns of the identity matrix $I = I_n$. The first of the two statements above simply expresses the fact that when we write a column vector as a simple list of numbers, those numbers are implicitly interpreted as **the coordinates relative to the standard basis**. The second statement above shows that different ordered bases lead to different coordinate vectors.