An Example in Polynomial Space

Ex: Let us take $V = P_3, W = P_2$ and let T be "differentiation". We will use bases $E = (1, t, t^2, t^3)$ in V and $E = (1, t, t^2)$ in W. We simply need to compute T applied to the first basis and express the results in terms of the second basis. Here it is:

$$
\begin{aligned}
T(1) &= (1)' = 0 = 0 \times 1 + 0 \times t + 0 \times t^2 \\
T(t) &= (t)' = 1 = 1 \times 1 + 0 \times t + 0 \times t^2 \\
T(t^2) &= (t^2)' = 2t = 0 \times 1 + 2 \times t + 0 \times t^2 \\
T(t^3) &= (t^3)' = 3t^2 = 0 \times 1 + 0 \times t + 3 \times t^2
\end{aligned}
$$

From this we read off the coordinate matrix:

$$
[T]_{EE} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}
$$