Example (continued)

Let us continue the example now by using instead the basis H in the range space. We need constants a_{ij} such that

$$T(e_1) = \begin{bmatrix} 3 \\ -1 \end{bmatrix} = a_{11}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_{21}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a_{12}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_{22}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$$

We can find the a_{ij} by a double G-E:

$$\left[\begin{array}{cc|cc} 1 & 1 & 3 & 1 \\ 1 & -1 & -1 & 0 \end{array}\right] \xrightarrow{\text{G-E}} \left[\begin{array}{cc|cc} 1 & 0 & a_{11} & a_{12} \\ 0 & 1 & a_{21} & a_{22} \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 2 & \frac{1}{2} \end{array}\right]$$

Thus

$$[T]_{HE} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{1}{2} \end{bmatrix}$$