Example (continued)

In W we could use either E or H as the basis. Consider first E. This means that we need to write $T(e_1)$ and $T(e_2)$ in terms of this basis. Then we can read off the coordinate matrix:

$$\begin{array}{l} T(e_1) = 3e_1 - 1e_2 \\ T(e_2) = 1e_1 + 0e_2 \end{array} \quad \text{and so} \quad [T]_{EE} = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$$

To check that we are getting things correct, we see that

$$[T(v)]_E = [T]_{EE}[v]_E \quad \text{becomes} \quad \begin{bmatrix} 3v_1 + v_2 \\ -v_1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

We saw previously that every linear transformation of the plane to itself was respresented by a matrix, in this case the one shown here. Therefore we could equally well write

$$[\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}]_{EE} = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$$

The matrix of a linear transformation of $\mathbb{R}^2$ to $\mathbb{R}^2$ is implicitly that relative to the standard basis in both domain and range space.