Coordinates of Vectors

Previously we saw that you can express a vector in terms of a basis in exactly one way. Let us formalize that idea now.

Definition: An ordered basis $F = (f_1, f_2, ..., f_n)$ of a vector space V is a basis whose members have been ordered from first to last in some way (as indicated here by the subscripts). Let v be a vector in V. Then there are **unique** coefficients $a_1, a_2, ..., a_n$ such that

$$v = a_1f_1 + a_2f_2 + \cdots + a_nf_n$$

We call the a_i 's the coordinates of v relative to the basis F and adopt the notation

$$[v]_F = \begin{bmatrix} a_1\\ \vdots\\ a_n \end{bmatrix}$$

Note: Even though a vector can be something rather exotic (a polynomial, a digital image, etc.), its coordinate vector relative to a basis **is always a column vector**.