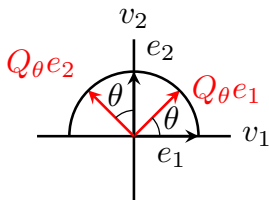


A Rotation Matrix

What matrix Q_θ represents (by multiplication) a θ radians counter-clockwise rotation in the plane \mathbb{R}^2 ?

We need to determine the value of multiplying Q_θ times the standard basis vectors e_1 and e_2 . Setting $c = \cos \theta$, $s = \sin \theta$, trigonometry gives us:



$$Q_\theta e_1 = Q_\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c \\ s \end{bmatrix}$$
$$Q_\theta e_2 = Q_\theta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s \\ c \end{bmatrix}$$

Therefore

$$\begin{aligned} Q_\theta v &= Q_\theta \left(v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = v_1 Q_\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 Q_\theta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= v_1 \begin{bmatrix} c \\ s \end{bmatrix} + v_2 \begin{bmatrix} -s \\ c \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} v \end{aligned}$$