

## A Variation on the Example

**Ex:** If  $T$  maps  $\mathbb{R}^2$  to  $\mathbb{R}^2$  and  $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$  and

$T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ , find a formula for  $T(v)$  for any  $v$ . Since (from previous calculations)

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{v_1 + v_2}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{v_1 - v_2}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

we have

$$\begin{aligned} T(v) &= \frac{v_1 + v_2}{2} T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) + \frac{v_1 - v_2}{2} T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) \\ &= \frac{v_1 + v_2}{2} \begin{bmatrix} -2 \\ 6 \end{bmatrix} + \frac{v_1 - v_2}{2} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2v_2 \\ 4v_1 + 2v_2 \end{bmatrix} \end{aligned}$$