

Constructing Linear Transformations

Ex: If T maps \mathbb{R}^2 to \mathbb{R}^2 and $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, find a formula for $T(v)$ for any v . Here is how we do it. Since

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

we have

$$\begin{aligned} T(v) &= v_1 T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + v_2 T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \\ &= v_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} + v_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -v_1 + 2v_2 \\ 3v_1 + 2v_2 \end{bmatrix} \end{aligned}$$

For example

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -(1) + 2(2) \\ 3(1) + 2(2) \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$