Linear Transformations on Basis Vectors

Ex Int: $V = P_2, W = P_3$ and

$$T(v(t)) = T(\underbrace{a_0 + a_1 t + a_2 t^2}_{\text{generic member of } P_2}) = \underbrace{a_0 t + \frac{1}{2}a_1 t^2 + \frac{1}{3}a_2 t^3}_{\text{in } P_3}$$

This is just integration of polynomials (with no constant term) and you learned in calculus that integration is linear! (an integral of a sum is the sum of the integrals, etc.)

Theorem: Let $T : V \to W$ be a linear transformation and $\{f_1, f_2, ..., f_n\}$ a basis of V. Then T is completely determined by its values on a basis, i.e. $Tf_1, Tf_2, ..., Tf_n$. Specifically, if v is any vector in V, then v is some linear combo of the basis vectors, say $v = v_1 f_1 + v_2 f_2 + \cdots + v_n f_n$. Then

$$T(v) = T(v_1 f_1 + \cdots + v_n f_n) = v_1 T(f_1) + \cdots + v_n T(f_n)$$

This result is very useful if either the values of T on a basis are given or if the values on a basis are easy to construct.