More Examples in 2D

This transformation is just a $90°$ counter-clockwise rotation. Is it linear? Yes:

$$T(c_1 u + c_2 v) = T(\begin{bmatrix} c_1 u_1 + c_2 v_1 \\ c_1 u_2 + c_2 v_2 \end{bmatrix}) = \begin{bmatrix} -c_1 u_2 - c_2 v_2 \\ c_1 u_1 + c_2 v_1 \end{bmatrix}$$

$$= c_1 \begin{bmatrix} -u_2 \\ u_1 \end{bmatrix} + c_2 \begin{bmatrix} -v_2 \\ v_1 \end{bmatrix} = c_1 T(u) + c_2 T(v)$$

Obvservation: Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be $T(v) = Av$ for some $m \times n$ matrix A. Is T linear? Yes:

$$T(c_1 u + c_2 v) = A(c_1 u + c_2 v) = c_1(Au) + c_2(Av) = c_1 T(u) + c_2 T(v)$$

That is, any mapping on column vectors defined by "multiplication by a matrix" is linear.